Algebraic Topology

M.Math II year

Mid-Semestral Examination

26.02.2007

1. Decide whether the following statements are TRUE or FALSE. Justify your answers. State precisely the results that you use.

   (a) The pull back of a trivial bundle is trivial.

   (b) The tangent bundle of $S^2$ splits as the Whitney sum of two line bundles.

   (c) $w(\gamma^1) = 1 + a$.

   (d) There exists a vector bundle $\eta$ such that $\gamma^1 \oplus \eta$ is trivial.

   (e) The manifolds $\mathbb{RP}^2 \times \mathbb{RP}^3$ and $S^5$ are cobordant.

2. Answer any *three* of the following.

   (a) Determine the least integer $k$ assuming that $\mathbb{RP}^9$ can be immersed in $\mathbb{R}^{9+k}$.

   (b) Show that the tangent bundle of $\mathbb{RP}^n$ has a sub bundle of rank 1 if and only if $n$ is odd.

   (c) Let $n = 2^k$. Find all the Stiefel-Whitney numbers of $\mathbb{RP}^n$.

   (d) Let $k \geq 1$ be an integer and $f_k : S^1 \longrightarrow S^1$ be the power map $z \mapsto z^k$. For which values of $k$ is the bundle $f_k^*(\gamma_1^1)$ trivial?